Accurate extraction of the News

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We propose a new scheme for extracting gravitational radiation from a characteristic numerical simulation of a spacetime. This method is similar in conception to our earlier work but analytical and numerical implementation is different. The scheme is based on direct transformation to the Bondi coordinates and the gravitational waves are extracted by calculating the Bondi news function in Bondi coordinates. The entire calculation is done in a way which will make the implementation easy when we use uniform Bondi angular grid at $\mathcal{I}^+$. Using uniform Bondi grid for news calculation has added advantage that we have to solve only ordinary differential equations instead of partial differential equation. For the test problems this new scheme allows us to extract gravitational radiation much more accurately than the previous schemes.

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I. INTRODUCTION

One of the main aims of the numerical relativity simulations of spacetimes is to calculate the gravitational radiation emitted by various kinds of sources, particularly in the strong field regime. The predicted waveforms for different kinds of astrophysical sources will be useful for detection of gravitational waves and parameter estimation of sources (by matched filtering technique) using many working and upcoming gravitational wave detectors like, LIGO, VIRGO, TAMA, GEO and LISA. For this we need the simulations to be very accurate and we also need to extract the gravitational waves faithfully and accurately from the simulations. Both these tasks are very challenging. Binary black holes and neutron stars are the main likely sources for early detection. In the recent times there is considerable progress in evolving the binary black hole [1, 2, 3, 4, 5, 6, 7, 8, 9] and binary neutron star [10] spacetimes, but a lot of improvements are needed for the predicted waveforms to be useful for the future observations.

In the most common approach to the numerical relativity (Cauchy/ADM approach) the spacetime is foliated by a sequence of spacelike hypersurfaces parameterized by time. In such a case we cannot extract the gravitational waves exactly and some approximate techniques have to be developed. These techniques [11, 12, 13, 14, 15, 16] use the data at the outer boundary of the simulations to predict the gravitational wave content of spacetimes at future null infinity ($\mathcal{I}^+$). In recent times new promising ways, which are also useful for extracting gravitational waves from Cauchy simulations of spacetimes have been proposed [17, 18, 19, 20, 21, 22, 23]. But in this paper we use the characteristic (null cone) formulation using the Bondi-Sachs coordinates. The spacetime is foliated by series of null hypersurfaces parameterized by outgoing (/ingoing) null geodesics. Using compactification techniques, $\mathcal{I}^+$ is included in the computational domain. In characteristic numerical simulations one can in principle extract the gravitational radiation exactly at null infinity ($\mathcal{I}^+$) using the Bondi news function [24, 25, 26]. Even then the accurate extraction of the news seems quite tricky as many numerical complications make the task very hard. The news function takes a very simple form in the Bondi coordinates, but the numerical simulations use more general Bondi-Sachs coordinates. The Bondi coordinates correspond to inertial observers at $\mathcal{I}^+$, but in a numerical code we want to provide boundary conditions at inner boundary (e.g. black hole horizon) and then solve the equations as we go out to $\mathcal{I}^+$, so we use the more general Bondi-Sachs coordinates. Earlier news was computed directly in Bondi-Sachs coordinates [27] and then a transformation was done to the Bondi coordinates (as detectors are more or less in the reference frame of inertial observer at $\mathcal{I}^+$). The news expression in Bondi-Sachs coordinates is quite complicated.

We had proposed a scheme [28] where we calculated the news by going directly to the Bondi coordinates. But further tests showed that improvement in the news calculations were needed. In certain cases numerically intermediate constraint, $\tilde{J}_0 = 0$ (it basically says, the tangential part of Bondi metric at $\mathcal{I}^+$ should be zero, the notation is introduced in Sec. II) was not getting satisfied, though as such the final news calculations gave satisfactory result.

The problem was traced to a particular (i.e. $\eth X$) term in equation (20) for $\tilde{J}_0$ in [28]. This term is basically related to an angular derivative of the Bondi angular coordinate with respect to the Bondi-Sachs angular coordinate. Many techniques, like doing entire numerical calculations using spin weighted spherical harmonic coefficients etc. were tried to improve the accuracy of the result while sticking to the original analytical procedure in [28], but it was hard to achieve the desired results.

The main reason for this problem seems to be the interaction of various numerical errors creating unexpected enhancement of error while calculating angular derivatives. Using the scheme in [28], these angular derivatives had to be calculated in a complicated way, using Jacobian of coordinate transformation. This was needed

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as all the angular derivatives were with respect to the Bondi-Sachs coordinates while for calculating the news we needed to use uniform Bondi angular grid. This choice had to made so that one could interpolate things more easily and also as then one needed to solve only ordinary differential equations for doing the coordinate transformation instead of partial differential equations.

In this paper we present a scheme which suites these numerical requirements better and gives the desired accuracy. The scheme is similar in conception to the scheme in [28], but the analytical and numerical implementation is different. The news is calculated by directly going to the Bondi coordinates. At the initial slice J_0 has to be zero (notation is introduced in section Sec. II) and the Bondi-Sachs and Bondi coordinates have to match at $\mathcal{I}^+$. For the news calculation we stick to the Bondi coordinates and uniform Bondi grid at $\mathcal{I}^+$. The transformation is done analytically (partly using Mathematica) so that the numerical implementation will have good accuracy. We have implemented our scheme and tested it with analytical solutions. The calculations and results are presented in this paper. This new scheme shows very good accuracy for the news extraction and also the constraints $\tilde{J}_0 = 0$ is satisfied with desired accuracy.

The paper starts with a summary of relevant results and notation for the characteristic formulation of numerical relativity (Sec. II). The coordinate transformation and algebraic calculations and expressions are given in Sec. III, and then the procedure for computing the news, at both analytic and computational levels, is described in Sec. IV. The computational tests and results are presented in Sec. V followed by discussions and conclusions in Sec. VI.

II. NOTATION

Here we briefly review the notation and formalism for the characteristic numerical relativity [27, 29] (see also [24, 30, 31, 32]). The formalism is based on a family of outing null hypersurfaces using the Bondi-Sachs coordinates. The hypersurfaces are labeled by u, x^A ($A = 2, 3$) label the null rays and r is a surface area coordinate. The metric in these Bondi-Sachs coordinates [24, 33] is written as,

$$\begin{aligned} ds^2 = &-\left(e^{2\beta}(1+\frac{W}{r}) - r^2 h_{AB}U^A U^B\right) du^2 \\ &-2e^{2\beta}dudr - 2r^2 h_{AB}U^B du dx^A + r^2 h_{AB} dx^A dx^B, \end{aligned} \quad (1)$$

where $h^{AB}h_{BC} = \delta^A_C$ and $det(h_{AB}) = det(q_{AB})$, with q_{AB} a unit sphere metric. We use stereographic coordinates $x^A = (q, p)$ for which the unit sphere metric is

$$q_{AB}dx^A dx^B = \frac{4}{P^2}(dq^2 + dp^2), \quad (2)$$

where

$$P = 1 + q^2 + p^2. \quad (3)$$

We introduce a complex dyad q_A defined by

$$q^A = \frac{P}{2}(1, i), \quad q_A = \frac{2}{P}(1, i) \quad (4)$$

with $i = \sqrt{-1}$. For an arbitrary Bondi-Sachs metric, h_{AB} can then be represented by its dyad component

$$J = h_{AB}q^A q^B/2. \quad (5)$$

h_{AB} is uniquely determined by J, since the determinant condition ($det(h_{AB}) = det(q_{AB})$) implies that there are only two independent components of h_{AB} and the remaining dyad component

$$K = h_{AB}q^A\bar{q}^B/2 \quad (6)$$

satisfies $1 = K^2 - J\bar{J}$. For spherically symmetric case J is identically zero. We introduce the spin-weighted field,

$$U = U^A q_A. \quad (7)$$

We also introduce complex angular coordinate $\zeta = q + ip$ as well as the (complex differential) eth operators $\eth$ and $\bar{\eth}$ (see [34] for full details) which are given as,

$$\eth A = P\frac{\partial A}{\partial\bar{\zeta}} + sA\zeta, \quad \bar{\eth}A = P\frac{\partial A}{\partial\zeta} - sA\bar{\zeta}, \quad (8)$$

where A is any spin weighted field with spin weight s.

The news is calculated in a conformally compactified coordinates. Specifically, $(u, r, x^A) \to (u, \ell, x^A)$ where $\ell = 1/r$. In (u, ℓ, x^A) coordinates, the compactified metric is $d\hat{s}^2 = \ell^2 ds^2$ where ℓ is a conformal factor with future null infinity $\mathcal{I}^+$ given by $\ell = 0$. The compactified metric will be denoted by $\hat{g}^{\alpha\beta}$ ($\alpha, \beta = 0-3$) and the general compactified Bondi-Sachs metric is

$$\begin{aligned} \hat{g}^{11} = e^{-2\beta}V_a, \quad \hat{g}^{1A} = e^{-2\beta}U^A, \\ \hat{g}^{10} = e^{-2\beta}, \quad \hat{g}^{AB} = h^{AB}, \quad \hat{g}^{0A} = \hat{g}^{00} = 0, \end{aligned} \quad (9)$$

where $V_a = \ell^2(1+\ell W)$. In addition to the general compactified Bondi-Sachs metric Eq. (9), we will refer to the compactified Bondi-Sachs metric satisfying the Bondi conditions, and such quantities will be denoted with a tilde (˜), with compactified metric and coordinates $\tilde{g}^{\alpha\beta}$ and $(\tilde{u}, \tilde{\ell}, \tilde{x}^A)$, respectively. On $\mathcal{I}^+$, i.e. $\tilde{\ell} = 0$, $\tilde{g}^{\alpha\beta}$ satisfies the Bondi conditions

$$\tilde{g}_{00} = 0, \quad \tilde{g}_{0A} = 0, \quad \tilde{g}_{01} = 1, \quad \tilde{g}_{AB} = \tilde{q}_{AB}, \quad (10)$$

where $\tilde{q}^{AB}$ is a unit sphere metric with respect to the Bondi angular coordinates $\tilde{x}^A$.

III. COORDINATE TRANSFORMATION

We define a coordinate transformation, near $\mathcal{I}^+$, between (u, ℓ, x^A) and $(\tilde{u}, \tilde{\ell}, \tilde{x}^A)$ as a Taylor series expansion in $\tilde{\ell}$. Up to second order in $\tilde{\ell}$ we can write,

$$\begin{aligned} u = u_0 + A^u\tilde{\ell} + C^u\tilde{\ell}^2, \quad \ell = \tilde{\ell}/\omega + C^\ell\tilde{\ell}^2, \\ x^A = x^A_0 + A^A\tilde{\ell} + C^A\tilde{\ell}^2, \end{aligned} \quad (11)$$

where ω, x_0^A, A^A, u_0, A^u are all functions of $\tilde{x}^A$ and $\tilde{u}$ only. It will turn out that the C^α are not needed for the news calculation, but they are included so that the Jacobian for coordinate transformation is manifestly correct to first order in $\tilde{\ell}$. We also introduce complex quantities

$$A = \tilde{q}_A A^A \text{ and } X = \tilde{q}_A \tilde{x}_0^A. \tag{12}$$

The metric functions are expanded to first order in $\tilde{\ell}$ as,

$$\beta = \beta_0 + \tilde{\ell}\beta_{\tilde{\ell}},\ \ U = U_0 + \tilde{\ell}U_{\tilde{\ell}},\ \ J = J_0 + \tilde{\ell}J_{\tilde{\ell}},\\ K = K_0 + \tilde{\ell}K_{\tilde{\ell}},\ \ \tilde{V}_a = \tilde{\ell}V_{a\tilde{\ell}}. \tag{13}$$

The same first-order expansion in $\tilde{\ell}$ and notation will be used for the metric quantities in the coordinate system satisfying the Bondi conditions (e.g., $\tilde{J} = \tilde{J}_0 + \tilde{\ell}\tilde{J}_{\tilde{\ell}}$). The general compactified Bondi-Sachs metric, and the Bondi-Sachs metric satisfying the Bondi conditions, are related by

$$(\tilde{g}_{\alpha\beta})_0 + \tilde{\ell}(\tilde{g}_{\alpha\beta})_{\tilde{\ell}} = \tilde{g}_{\alpha\beta} = \omega^2 \frac{\partial x^\mu}{\partial \tilde{x}^\alpha}\frac{\partial x^\nu}{\partial \tilde{x}^\beta}\hat{g}_{\mu\nu}, \tag{14}$$

with the factor ω^2 appearing because there is also an implicit change of compactification factor from ℓ to $\tilde{\ell}$.

We use Mathematica to find $\tilde{g}^{\alpha\beta}$ to first order in $\tilde{\ell}$, and in doing so we have imposed the conditions,

$$\frac{\partial u}{\partial \tilde{u}} = e^{-2\beta_0} \tag{15}$$

and

$$\frac{\partial x^A}{\partial \tilde{u}} = U^A e^{-2\beta}/\omega_0. \tag{16}$$

These conditions are equivalent of satisfying,

$$(\partial_u + U_0^B \partial_B)\tilde{x}_0^A = 0 \tag{17}$$

and

$$(\partial_u + U_0^B \partial_B)\tilde{u}_0 = \omega e^{2\beta_0}, \tag{18}$$

along the generators of $\mathcal{I}^+$ (see Eqs. (39) and (40) in [27] for some details). This ensures Bondi conditions $\tilde{g}_{00} = 0$, $\tilde{g}_{0A} = 0$, $\tilde{g}_{01} = 1$ are satisfied identically.

We also evolve ω along the generators of $\mathcal{I}^+$ by integrating [27],

$$\frac{d\omega}{du} = -\frac{1}{4}\omega(\eth U + \bar{\eth}\bar{U}). \tag{19}$$

Evolving ω using this ordinary differential equation reduces the errors due to angular derivatives as compared to the method used in [28]. Knowing the coordinate transformation and ω, we can find the value of $\tilde{J}_0$ (J_0 in Bondi coordinates),

$$\begin{aligned}\tilde{J}_0 \;=\; & -\frac{\omega^2}{4P^2}((\eth\bar{X} + B_X\tilde{P}\bar{X})^2 J_0\tilde{P}^2 + (\eth X)^2 \bar{J}_0\tilde{P}^2 \\ & + P^2(\eth u)^2(J_0\bar{U}_0^2 + 2K_0\bar{U}_0U_0 + \bar{J}_0{U_0}^2) \\ & -2P\tilde{P}\eth u(J_0\bar{U}_0 + K_0U_0)(\eth\bar{X} + B_X\tilde{P}\bar{X}) \\ & -2P\tilde{P}\eth u\eth X(K_0\bar{U}_0 + \bar{J}_0U_0) \\ & +2\tilde{P}^2K_0\eth X(\eth\bar{X} + B_X\tilde{P}\bar{X})),\end{aligned} \tag{20}$$

Where, $B_X = \frac{2\tilde{\zeta}}{\tilde{P}}$. The analytical value of $\tilde{J}_0$ has to be zero as we are in Bondi coordinates and we can use the numerical value to estimate the accuracy of the coordinate transformation.

Bondi-Sachs conditions

$$\hat{g}^{00} = 0, \tag{21}$$

gives us from the lowest (0^{th}) order in $\tilde{\ell}$,

$$A_u = -\frac{1}{2}\omega e^{2\beta}\eth u\bar{\eth}u. \tag{22}$$

First order in $\tilde{\ell}$ will give us C_u but, it is not needed for news calculation. Bondi-Sachs condition,

$$\hat{g}^{0A} = 0, \tag{23}$$

leads to,

$$A = -\frac{\omega e^{2\beta}}{2\tilde{P}}(((\bar{\eth}X + \bar{B}_X\tilde{P}X)\tilde{P} - \bar{\eth}uU_0P)\eth u + \bar{\eth}u\eth X\tilde{P}) \tag{24}$$

from the lowest (0^{th}) order in $\tilde{\ell}$, and will give C from 1^{st} order, but it is not needed for our calculation. One can also get A_u and A by solving for $\tilde{g}_{11} = 0$ and $\tilde{g}_{0A} = 0$ but we use $\hat{g}^{00} = 0$, $\hat{g}^{0A} = 0$ as it simplifies the calculation.

After knowing these quantities, we can calculate $\tilde{J}_{\tilde{\ell}}$. The expression for $\tilde{J}_{\tilde{\ell}}$ is quite long and is given in the equation (A1) in the appendix.

IV. CALCULATING THE NEWS

After knowing the coordinate transformations to required order we can develop a systematic scheme for extracting the news. First we describe the analytic aspects and then we discuss some details about the actual implementation in the program. It is assumed that the Bondi-Sachs metric is knows throughout the spacetime.

A. Procedure

1. We integrate equation (16) to get x^A on a fixed Bondi grid.
2. We evolve equation (19) for ω.
3. $\tilde{J}_0$ is evaluated form equation (20) and is used to monitor the numerical accuracy of the calculation.
4. We find the time transformation using equation (15).
5. A_u is calculated using equation (22) and A from equation (24).
6. We evaluate $\tilde{J}_{\tilde{\ell}}$ using equation (A1).

7. The news is extracted by differentiating $\tilde{J}_{\tilde{\ell}}$,

$$N = \frac{1}{2}\frac{\partial^2 \tilde{J}}{\partial \tilde{\ell}\partial \tilde{u}} = \frac{1}{2}\frac{\partial \tilde{J}_{\tilde{\ell}}}{\partial \tilde{u}} \tag{25}$$

B. Computational implementation

The news module has been written to interface directly with the null gravity code in its current form [35]. Thus we use a compactified radial coordinate $x = r/(R+r)$, with $R = 1$. There are n_x points in the x direction in the range [0.5, 1] (corresponding to $1 < r < \infty$). We use stereographic angular coordinates with two patches with minimum patch overlap of 5 grid points. Both patches have n_n grid points in each angular direction.

In actual numerical implementation for the news calculation, we use uniform $(\tilde{u}, \tilde{\ell}, \tilde{x}^A)$ grid instead of uniform $(u, \tilde{\ell}, \tilde{x}^A)$ to avoid having interpolation with respect to time. This means in place of equation (15) we solve its inverse,

$$\frac{d\tilde{u}}{du} = \omega e^{2\beta}, \tag{26}$$

along the generators of $\mathcal{I}^+$. This also means that in the derivative operators with Bondi coordinates, have to be replaced by,

$$\frac{\partial}{\partial \tilde{u}} \rightarrow \frac{e^{-2\beta_0}}{\omega}\frac{\partial}{\partial u} \tag{27}$$

and

$$\frac{\partial}{\partial \tilde{x}^A}|_{\tilde{u}} \rightarrow \frac{\partial}{\partial \tilde{x}^A}|_u + \frac{e^{-2\beta_0}}{\omega}\frac{\partial \tilde{u}}{\partial \tilde{x}^A}|_u \frac{\partial}{\partial u}. \tag{28}$$

We also introduce $\bar{\eth}U$ and $\eth U$ as well as $\bar{\eth}X$ and $\eth X$ as additional variables. This helps to reduce the numerical errors as typically news indirectly involves 3^{rd} or higher angular derivative of U and X (if the original coordinates are not already Bondi coordinates). Higher angular derivatives are known to give lot of problems numerically. Introducing these additional variables is one simple way of improving the numerical behavior. These variables should be introduced in the null evolution code as well. All integration and differentiation schemes are 2^{nd} order, apart from for the first step, for which time integration is done by 1^{st} order scheme.

V. COMPUTATIONAL TESTS AND RESULTS

We implemented and tested our method for calculating the Bondi news function for Schwarzschild and linearized Robinson-Trautman in rotating as well as tumbling coordinates. The analytical solution on the discretized grid is given as the input to the news module. The news module as such sees it as input from an equivalent numerical evolution code. The same news module can be used with suitable null evolution code.

A. Schwarzschild solution in rotating and tumbling coordinates

We get the Schwarzschild solution in Bondi-Sachs rotating coordinates by transforming the standard angular coordinates by, $\phi \rightarrow \tilde{\phi} + \kappa u$. Various metric coefficients at $\mathcal{I}^+$ are written in this case as,

$$\begin{aligned} W_N = W_S = 0, \quad & \beta_N = \beta_S = 0, \\ J_N = J_S = 0, \quad & (J_\ell)_N = (J_\ell)_S = 0, \\ U_N = -\frac{2i\kappa\zeta_N}{M(1+\zeta_N\bar{\zeta}_N)}, \quad & U_S = \frac{2i\kappa\zeta_S}{M(1+\zeta_S\bar{\zeta}_S)}, \end{aligned} \tag{29}$$

where the subscripts N and S refer to north and south patch.

When we have coordinate system which is rotating around an equatorial axis, we call it a tumbling coordinate system. In this case the relationship of the tumbling angular coordinates to the Bondi angular coordinates is given as,

$$\tilde{\theta} = \arccos(\cos\theta\cos\kappa u + \sin\theta\sin\phi\sin\kappa u) \tag{30}$$

$$\tilde{\phi} = \arctan\left(\frac{\cos\kappa u\sin\theta\sin\phi - \cos\theta\sin\kappa u}{\sin\theta\cos\phi}\right) \tag{31}$$

In these Bondi-Sachs coordinates only nonzero part of the metric at $\mathcal{I}^+$ is U, which is given as,

$$\begin{aligned} U_N &= i\kappa\frac{1-\zeta_N{}^2}{(1+\zeta_N\bar{\zeta}_N)}, \\ U_S &= i\kappa\frac{1-\zeta_S{}^2}{(1+\zeta_S\bar{\zeta}_S)}. \end{aligned} \tag{32}$$

The convergence tests were performed for the following grid sizes for all the tests,

$$\begin{aligned} &(a) \quad \Delta q = \Delta p = 1/8, \Delta x = 1/64, \Delta u = 0.04 \\ &(b) \quad \Delta q = \Delta p = 1/16, \Delta x = 1/128, \Delta u = 0.02 \\ &(c) \quad \Delta q = \Delta p = 1/32, \Delta x = 1/256, \Delta u = 0.01 \\ &(d) \quad \Delta q = \Delta p = 1/64, \Delta x = 1/64, \Delta u = 0.005. \end{aligned} \tag{33}$$

For the highest resolution run we keep $\Delta x = 1/64$ due to memory limitations, but as such the results do not change with Δx.

In both these cases the L_2 norm of $\tilde{J}_0$ shows 2nd (1.99) order convergence to zero as in figure (1) and the convergence rate is essentially independent of u.

The news function with our scheme remains identically zero. This is an ideal behavior and is a pleasant surprise. The news shows this behavior as A and A_u are zero analytically and with our scheme and choice of variables they also remain zero numerically. From the expression for $\tilde{J}_\ell$ one can see that it will be zero for our case. Due to

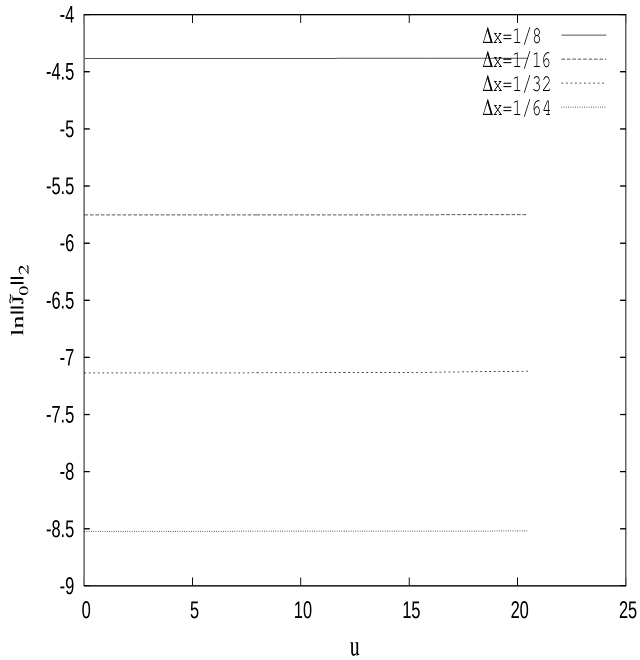

FIG. 1: L_2 norm of $||\tilde{J}_0||_2$ for Schwarzschild in tumbling coordinates as function of u. The norm changes very slowly with u. Analytically $\tilde{J}_0$ should be zero. The graph for Schwarzschild in rotating coordinates looks essentially identical, so we have not plotted it separately.

this we can expect to extract out news accurately even if we happen to be in a rotating or tumbling kind of background coordinates and the amplitude of gravitational radiation is very small.

The scheme in [28] showed required convergence for $\tilde{J}_0$ in rotating Schwarzschild case, but later it was realized it had problems while dealing with the tumbling Schwarzschild case. It was very hard to remove these numerical problems in that scheme. Also the error in the news in both these cases, though, convergent to 2^{nd} order, they were not all that small, if one had to extract very weak gravitational wave signals in nontrivial coordinates. Both these issues lead us to develop the scheme presented above.

B. Linearized Robinson-Trautman solution

The Robinson-Trautman solution represents a distorted black hole emitting purely outgoing radiation. The radiation decays exponentially, and asymptotically the solution becomes Schwarzschild. In the linearized case when the amplitude of the perturbation (i.e. also the gravitational radiation) is small one can write it as addition of different spherical harmonics (Y_{lm}) of various amplitudes (see e.g. [28]).

convergence rate at u	grids (a) & (b)	(b) & (c)	(c) & (d)
4	2.012	2.002	2.008
8	2.028	2.021	2.005
12	2.070	2.013	2.005
16	2.097	2.012	2.005
20	2.086	2.011	2.005

TABLE I: Change of convergence rate with u for the L_2 norm of error in the news function for linearized Robinson-Trautman in rotating coordinates for different grid resolutions in equation 33.

1. *Rotating coordinates*

The metric components for the linearized Robertson Trautmann solution, when only Y_{20} term is present (i.e. only the $l = 2$, $m = 0$ term of spherical harmonics is present in the perturbation) can be written at $\mathcal{I}^+$ as [36],

$$
\begin{aligned}
W_N = W_S &= 0, \\
\beta_N = \beta_S &= 0, \\
J_N = J_S &= 0, \\
(J_\ell)_N &= -12\lambda_{20} e^{-2u/M} M \frac{\zeta^2{}_N}{(1+\zeta_N\bar{\zeta}_N)^2}, \\
(J_\ell)_S &= -12\lambda_{20} e^{-2u/M} M \frac{\zeta^2{}_S}{(1+\zeta_S\bar{\zeta}_S)^2}, \\
U_N &= -\frac{2i\kappa\zeta_N}{M(1+\zeta_N\bar{\zeta}_N)}, \\
U_S &= \frac{2i\kappa\zeta_S}{M(1+\zeta_S\bar{\zeta}_S)},
\end{aligned} \tag{34}
$$

where the subscripts N and S refer to north and south patch, M gives the mass of the remnant black hole and λ_{20} is the initial (at $u = 0$) amplitude of the perturbation.

In this case $\tilde{J}_0$ shows 2^{nd} order convergence to zero. Basically the $\tilde{J}_0$ part remains same as for the corresponding Schwarzschild case (Fig. 1). The news also shows 2^{nd} order convergence to the analytical value on the Bondi grid given by,

$$
N = -12\lambda_{20} e^{-2u} \frac{\tilde{\zeta}^2}{(1+\tilde{\zeta}\bar{\tilde{\zeta}})^2}, \tag{35}
$$

on both the patches.

For our test in figure (2) we set $M = 1$, $\lambda_{20} = 3\times 10^{-7}$ and $\kappa = 0.01$. Even for very small or very hight value of λ_{20}, news shows 2^{nd} order convergence though the gravitational wave amplitude goes down by more than 10 order of magnitude as the time passes. The convergence rate are shown in table (II). This is much better behavior than we could get by the earlier news modules.

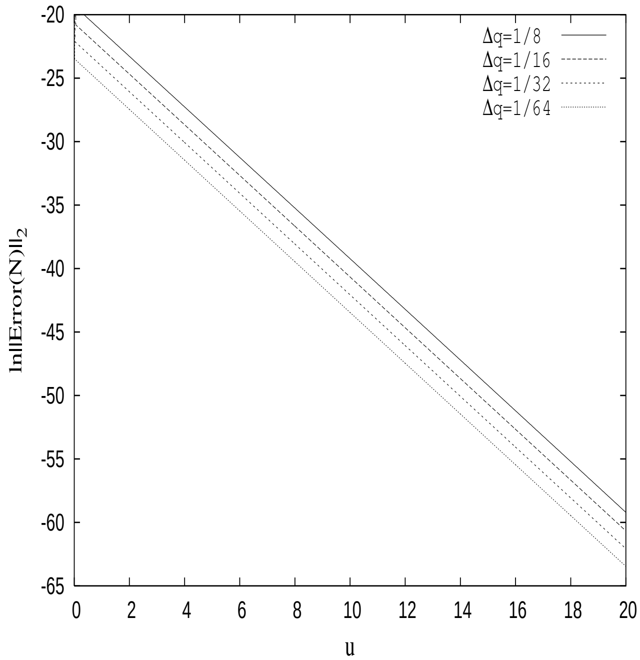

FIG. 2: L_2 norm of the error in the news as a function of time u for various discretizations for the linearized Robinson-Trautman solution in rotating coordinates.

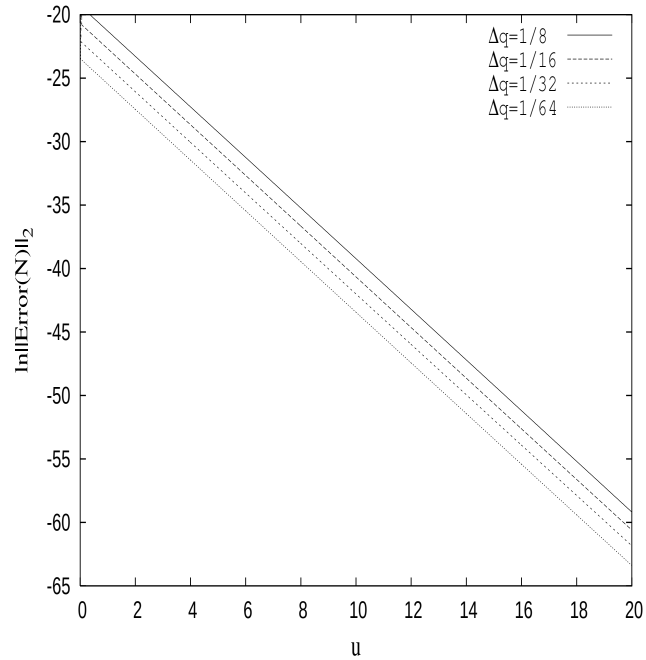

FIG. 3: L_2 norm of the error in the news as a function of time u for various discretizations for the linearized Robinson-Trautman solution in tumbling coordinates.

2. Tumbling coordinates

Various metric components for the linearized Robertson Trautmann solution, when only Y_{20} perturbation term is present can be written at $\mathcal{I}^+$ in tumbling coordinates [36] as,

$$\begin{aligned} W_N &= W_S = 0, \\ \beta_N &= \beta_S = 0, \\ J_N &= J_S = 0, \\ (J_\ell)_N &= 3M\lambda_{20}e^{-2u/M}\frac{2i\zeta_N\cos\kappa u + (1+\zeta_N{}^2)\sin\kappa u)^2}{(1+\zeta_N\bar{\zeta}_N)^2}, \\ (J_\ell)_S &= 3M\lambda_{20}e^{-2u/M}\frac{2i\zeta_S\cos\kappa u + (1+\zeta_S{}^2)\sin\kappa u)^2}{(1+\zeta_S\bar{\zeta}_S)^2}, \\ U_N &= i\kappa\frac{1-\zeta_N{}^2}{(1+\zeta_N\bar{\zeta}_N)}, \\ U_S &= i\kappa\frac{1-\zeta_S{}^2}{(1+\zeta_S\bar{\zeta}_S)}. \end{aligned} \tag{36}$$

In this case we again set $M = 1$, $\lambda_{20} = 3\times 10^{-7}$ and $\kappa = 0.01$. With this data, $\tilde{J}_0$ shows 2^{nd} order convergence to zero as for the Schwarzschild in tumbling coordinates. L_2 norm of the error in the news also shows 2^{nd} order convergence to zero as shown in figure (3) and table (II). The accuracy of news extraction remains very good as long as equator of one patch in Bondi coordinates doesn't go over the pole of the other patch in Bondi-Sachs coordinates due to tumbling. As such the accuracy of the news extraction does not depend on the initial amplitude of the waveform even it is increased or decreased by more that 10 orders of magnitude.

convergence rate at u	grids (a) & (b)	(b) & (c)	(c) & (d)
4	2.015	2.002	2.002
8	2.036	1.982	2.032
12	2.055	1.943	2.064
16	2.049	1.906	2.106
20	2.031	1.862	2.156

TABLE II: Change of convergence rate with u for the L_2 norm of error in the news function for linearized Robinson-Trautman in tumbling coordinates for different grid resolutions in equation 33.

VI. CONCLUSIONS

We have developed a new scheme for extracting the news from characteristic numerical simulations of the spacetimes. Like our earlier scheme this scheme is based on a coordinate transformation from general Bondi-Sachs coordinates to Bondi coordinates. But in the present scheme the transformation is done in a different way. It is done so that we can easily use a uniform and constant Bondi angular grid for the news calculation. This has an advantage that while transforming coordinates one has to evolve ordinary differential equations instead of partial ones. Also the interpolation and angular differentiation becomes easier and more accurate. We also introduce $\eth U$ and $\bar{\eth} U$ as auxiliary variables in the code to overcome problems related to numerical differentiations.

Like earlier schemes the present scheme also needs $J = 0$ at $\mathcal{I}^+$ on the initial time ($u = 0$) slice. It also needs the Bondi-Sachs and Bondi coordinates to match on the initial slice. We can also check the accuracy of coordinate transformation by monitoring values of $\tilde{J}_0$, which should be zero analytically.

Our new scheme shows 2^{nd} order convergence of $\tilde{J}_0$ to zero and overcomes the convergence problem of $\tilde{J}_0$ faced by our earlier scheme for the Schwarzschild solution in tumbling coordinates. Also with this scheme the news for Schwarzschild solution in rotating/tumbling coordinates stays exactly zero, while with the earlier scheme it is not so small value which was converging to zero to 2^{nd} order in grid size. In other cases also our new scheme shows 2^{nd} order convergence in extracting the news. For the Robinson-Trautman test, it successfully extracts out the news even when the amplitude of the gravitational waves is very small and we are in a coordinate system which is rotating/tumbling with respect to the Bondi coordinates. The news calculation remains accurate even though the signal goes down many orders of magnitude with time. The relative accuracy of the news extraction does not depend on the initial amplitude of the waveform even when it is increased or decreased by more that 10 orders of magnitude from the cases studied in this paper. This indicates our scheme gives very accurate and faithful results and the scheme is quite robust.

Introducing $\eth U$ and $\bar{\eth} U$ as auxiliary variables seems to play a crucial role in getting the news accurately. In most cases, whichever scheme we may choose to calculate the news, it will indirectly contain at least third angular derivatives of U. Higher angular derivatives are known to create problems in the null code. To take care of this problem earlier three complex variables corresponding to $\eth\beta$, $\bar{\eth} J$ and $\eth K$ were introduced in [37]. In a similar manner we suggest to introduce eth derivatives of U as new variables. It will be particularly useful when one is extracting gravitational waves from the simulations. All the tests we have done show significant improvement if we introduce these new variables.

The accuracy of our new scheme is very encouraging and we expect that the present method will be useful for extracting gravitational radiation from more realistic physical systems/simulations. It could play an important role in Cauchy-Characteristic extraction. We plan to explore this direction in a future work.

APPENDIX A: EQUATION FOR $\tilde{J}_\ell$

The expression J_ℓ can be written as,

$$
\begin{aligned}
\tilde{J}_\ell \; = \; & (-4\eth u K_0 \omega((\eth X(\bar{J}_0 - K_0)\tilde{P} + (-J_0 + K_0)\tilde{P}(\eth\bar{X} + \tilde{P}B_X\bar{X}) + 2iP\eth u \mathcal{I}m(\bar{U}_0(J_0 - K_0)) \\
& + P\eth u K_0 U_0)(-2P(U_\ell - \bar{U}_\ell + A_u\omega(U_u - \bar{U}_u)) + 2i\mathcal{I}m(\bar{A}\omega\tilde{P}(-\eth U + \eth\bar{U} + \tilde{P}\bar{U}_0 X))) \\
& + (-\tilde{P}(J_0 + K_0)(\eth\bar{X} + \tilde{P}B_X\bar{X}) - \tilde{P}\eth X(\bar{J}_0 + K_0) + 2P\eth u \mathcal{R}e(\bar{U}_0(J_0 + K_0))) \\
& ((2P((U_\ell + \bar{U}_\ell) + A_u\omega(U_u + \bar{U}_u))) + 2\mathcal{R}e(\bar{A}\omega\tilde{P}(\eth U + \eth\bar{U} + \tilde{P}\bar{U}_0 X)))) \\
& - (8\omega((\eth\bar{X} + B_X\tilde{P}\bar{X})^2 K_0\tilde{P}^2 + P^2(\eth u)^2\bar{U}_0(K_0\bar{U}_0 + \bar{J}_0 U_0) + (\eth\bar{X} + B_X\tilde{P}\bar{X})\tilde{P}^2\eth X\bar{J}_0 \\
& - \tilde{P}P\eth u(\eth\bar{X} + B_X\tilde{P}\bar{X})(2K_0\bar{U}_0 + \bar{J}_0 U_0) + P\eth u\tilde{P}(-\eth X\bar{J}_0\bar{U}_0))(P(J_\ell + A_u J_{0,u}\omega) \\
& + \frac{\omega\tilde{P}}{4}(\bar{A}(2\eth J + (1-i)J_0\tilde{P}\bar{X} - (1+i)J_0\tilde{P}X) + A(2\bar{\eth} J + (1+i)J_0\tilde{P}\bar{X} - (1-i)J_0\tilde{P}X))))/P \\
& - (8\omega((\eth X)^2 K_0\tilde{P}^2 + (\eth\bar{X} + B_X\tilde{P}\bar{X})J_0\tilde{P}(\eth X\tilde{P} - P\eth u U_0) + P\eth u U_0(P\eth u(J_0\bar{U}_0 + K_0 U_0)) \\
& + \eth X(P\eth u\tilde{P}(-J_0\bar{U}_0 - 2K_0 U_0)))(P(\bar{J}_\ell + A_u\bar{J}_{0,u}\,\omega) + \frac{\omega\tilde{P}}{4}(A(2\bar{\eth}\bar{J} - (1+i)\bar{J}_0\tilde{P}\bar{X} + (1-i)\bar{J}_0\tilde{P}X) \\
& + \bar{A}(2\eth\bar{J} - (1-i)\bar{J}_0\tilde{P}\bar{X} + (1+i)\bar{J}_0\tilde{P}X))))/P + 16K_0(P^2\eth u(2e^{2\beta}\eth\omega - \eth A_u\omega^2(J_0\bar{U}_0^2 + U_0(2K_0\bar{U}_0 + \bar{J}_0 U_0))) \\
& - \omega^2\tilde{P}^2((\eth\bar{A} + \bar{A}B_X\tilde{P})(\eth\bar{X}J_0 + B_X J_0\tilde{P}\bar{X} + \eth X K_0) + \eth A(\eth X\bar{J}_0 + \eth\bar{X}K_0 + B_X K_0\tilde{P}\bar{X})) \\
& + P\omega^2\tilde{P}((\eth u(\eth\bar{A} + B_X\tilde{P}\bar{A})(J_0\bar{U}_0 + K_0 U_0) + \eth A(K_0\bar{U}_0 + \bar{J}_0 U_0)) + \eth A_u((\eth\bar{X} + B_X\tilde{P}\bar{X})(J_0\bar{U}_0 + K_0 U_0) \\
& + \eth X(K_0\bar{U}_0 + \bar{J}_0 U_0))) + P^2 e^{2\beta}(\eth u)^2\omega V_{a\ell}))/(16P^2 K_0)
\end{aligned}
\tag{A1}
$$

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